MATH 141: Midterm 1

Name: ______________________________

Directions:

- Show your thought process (commonly said as ”show your work”) when solving each problem for full credit.
- If you do not know how to solve a problem, try your best and/or explain in English what you would do.
- Good luck!

Problem	Score	Points
1		10
2		10
3		10
4		10
5		10
6		10
7		10
		70

1. If

$$f(x) = x^2 - x \qquad g(x) = 3x^2 - x + 1 \qquad h(x) = \sin(x) \qquad j(x) = 2^x$$

Evaluate, expand, and/or simplify the following:

(a) $h\left(\frac{\pi}{6}\right)$

(b) $j(1) \cdot h(0)$

(c) $f(x) \cdot g(x)$

(d) $f(x+h) - f(x)$

2. Short answer questions:

   (a) Explain in English the intuition (not the definition) behind the symbols $\lim\limits_{x\to a} f(x) = L$.

   (b) True or false: We can simplify

$$\frac{3(x-2)^2(x+3) - 4(x+2)(x-3)^2}{5x(x-3)^2(x-2) - 4(x+3)}$$

   by crossing out the $x+3$.

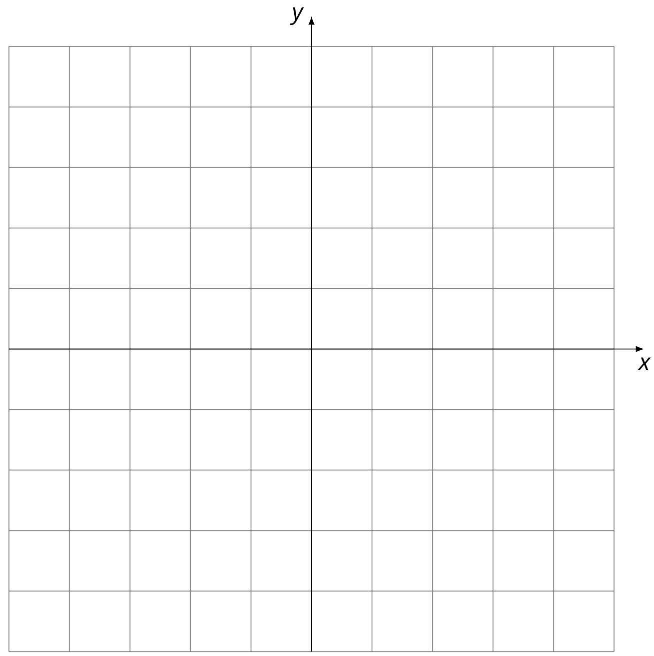
   (c) If $f(x) = x - x^2$, evaluate $f(x+h)$ and fully expand + simplify.

   (d) If $F(x) = \sin^3(x^2)$ find three functions f, g, h where $f \circ g \circ h = F$.

3. Suppose

$$f(x) = \begin{cases} x & x < 1 \\ -x^2 + 1 & x \geq 1 \end{cases}$$

(a) Sketch a graph of $f(x)$.



(b) What is $f(1)$?

(c) Does $\lim_{x \to 1} f(x)$ exist? If it does, find the limit. If not, explain why.

4. Perform the given instruction. Remember to use the relevant laws/properties and **fully simplify**.

   (a) Expand and simplify: $\dfrac{3(x+h)^2 - 1 - (3x^2 - 1)}{h}$

   (b) Rationalize the numerator (remember to simplify): $\dfrac{\sqrt{x+h} - \sqrt{x}}{h}$

(c) Simplify: $\dfrac{\dfrac{2}{x^2+x} - \dfrac{3}{\sqrt{x}}}{\sqrt{x} + \dfrac{1}{x}}$

(d) Expand: $(x^3 + 6)(2x + 1) - (x^2 + x - 2)(3x^2)$

5. Draw the graph of a function which satisfies the following:

   (a) $f(0) = 1$

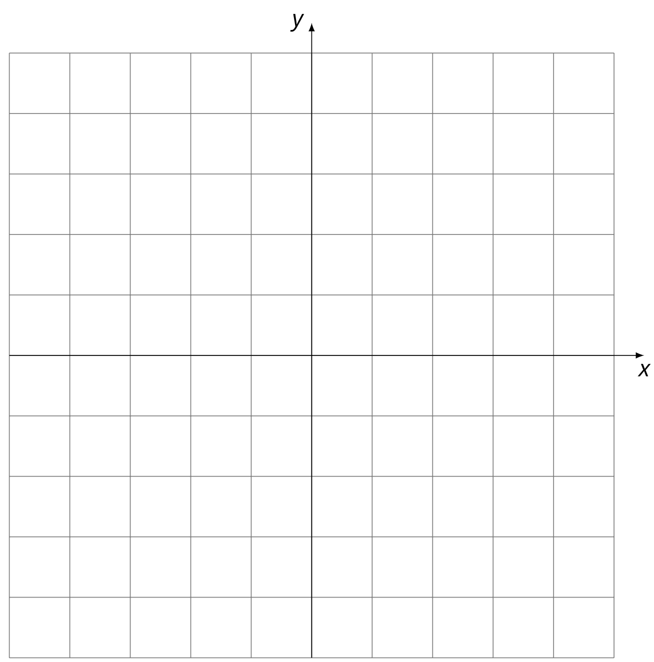
   (b) $f(2) = 1$

   (c) $\lim_{x\to 0} f(x) = 1$

   (d) $\lim_{x\to 2^-} f(x) = 0$

   (e) $\lim_{x\to 2^+} f(x) = 2$

   (f) $\lim_{x\to -2} f(x) = -\infty$



6. Consider this limit:

$$\lim_{h \to 0} \frac{\dfrac{1}{3+h} - \dfrac{1}{3}}{h}$$

(a) Try using Limit Laws to find the limit. What ends up happening?

(b) Now find the actual limit.

7. Use the **mathematical definition of continuity** to prove the function

$$f(x) = \begin{cases} x(x-1) & x < 1 \\ 0 & x = 1 \\ \sqrt{x-1} & x > 1 \end{cases}$$

is continuous at the number $x = 1$.